Higher Check In - 1.02 Whole number theory

Do not use a calculator for questions 1-5.

1. Write down a multiple of three that is between 123 345 and 123 445.

2. Find the Lowest Common Multiple (LCM) and Highest Common Factor (HCF) of 336 and 360. Give your answers as a product of prime factors.

3. Find the square root of 1600.

4. List the prime numbers that are also factors of 42.

5. Find the factors of 143.

6. Jenny says $G = 3^a \times 5^b$, where a and b are positive integers.
   Billy says $G = 2^t \times 3^a$, where t and a both positive integers.
   Can Billy and Jenny both be right? Explain your answer.

7. $a = 4^2 \times 6^3$ and $b = 2^3 \times 9^2$.
   Virgil says, “The numbers are written as products of factors using index form. As all the factors are different, a and b have no common factors”. Explain why Virgil is wrong and, without finding a and b, find their HCF.

8. For two numbers, c and d, the HCF = 1 and the LCM = cd. What can you say about c and d? Explain your answer and use examples to support this.

9. For two numbers, the HCF is 15 and the LCM is 210. Find all possible pairs of the two numbers.

10. These are the first four numbers that are each a square number **and also** a cube number.

	1	2	3	4
Square and cube number	1	64	729	4096

    Find the fifth number in the sequence.

Extension

What is the Lowest Common Multiple of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12?

Answers

1. Any number in range $123\,345 < n < 123\,445$ where the digit sum is a multiple of three e.g. 123 396.

2. HCF $= 2^3 \times 3$, LCM $= 2^4 \times 3^2 \times 5 \times 7$

3. $\sqrt{1600} = 40$

4. 2, 3 and 7

5. 1, 11, 13 and 323

6. No **AND** 2 is a factor of Billy's number so it must be even. Jenny's number does not have an even factor and so it must be odd. An even cannot equal an odd.
   OR It is only possible to write a number as a product of its prime factors in one way. These factors are all prime, so it is not possible for a number to be written as a product of sets of different primes.

7. The factors are not PRIME factors. As prime factors, $a = 2^2 \times 2^2 \times 2^3 \times 3^3$ (or $2^7 \times 3^3$) and $b = 2^3 \times 3^2 \times 3^2$ (or $2^3 \times 3^4$). This makes their HCF $2^3 \times 3^3$ or 216.

8. They cannot have a common factor, except 1. If, when written as the product of prime factors, a factor is common then it will appear in each list and the HCF will equal the common factor.
   14 and 21: $14 = 2 \times$ **7** and $21 = 3 \times$ **7** so the HCF = 7 and the LCM = $2 \times 3 \times 7 = 42$
   35 and 12: $35 = 5 \times 7$ and $12 = 2 \times 2 \times 3$ so the HCF = 1 and the LCM is $2 \times 2 \times 3 \times 5 \times 7 = 420$

9. $15 = 3 \times 5$ and $210 = 2 \times 3 \times 5 \times 7$. Possible pairs are 15 and 210 or 30 and 105

10. Sequence is $\left(1^2\right)^3, \left(2^2\right)^3, \left(3^2\right)^3, \left(4^2\right)^3$ so fifth term $\left(5^2\right)^3 = 15\,625$

Extension

$1 \times 2 \times 3 \times 2 \times 5 \times 7 \times 2 \times 3 \times 11 = 27\,720$

GCSE (9–1)

MATHEMATICS

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AO1	1	Understand and use multiples and place value			
AO1	2	Find the LCM of two numbers using prime factorisations			
AO1	3	Find square root			
AO1	4	Find prime numbers and factors of a given number			
AO1	5	Find all the factors of a number			
AO2	6	Understand that each number can be expressed as a product of prime factors in only one way			
AO2	7	Understand that each number can be expressed as a product of prime factors in only one way and find the HCF			
AO2	8	Use HCF and LCM of two whole numbers to explore the properties of the numbers			
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